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Moment Gradient Factor Verification for Selected Monosymmetric Beams under Linear Moment Gradients

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ABSTRACT

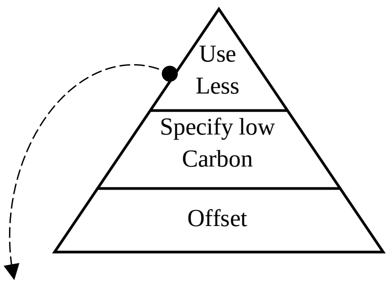
The reuse of doubly symmetric beams by converting them into monosymmetric section beams offers some promising outcomes and has potential for mitigating carbon footprint of structures. However, due to their complexity monosymmetric sections must be used in a configuration that allows the monosymmetry effects to act beneficially. Although the South African design standard for hot-rolled steel does not provide any guidance on the design of monosymmetric beams, the Southern African steel construction handbook provides a formula for determining the critical elastic buckling moment for monosymmetric beams. This guidance implies that the moment gradient factor used for doubly symmetric sections can be used on monosymmetric sections as well. The aim of the study was to verify the validity of this approach of extending the moment gradient factor used for doubly symmetric beams to monosymmetric beams for two specific types of monosymmetric sections. It was found that although this approach appears to be justified for monosymmetric members in single curvature bending it may produce unconservative values of the critical buckling load in double curvature bending between restraint points. The level of un-conservatism also varies for different spans of the same member. This makes it difficult to specify a single moment modification factor value for these cases. The sensitivity in terms of load reduction observed for double curvature bending case was different for the two members examined with this attributed to differences in how the shear centre moves relative to the centroid. It is recommended that the critical buckling load for monosymmetric sections be determined on a case specific basis for members in double curvature from linear moment gradients. Under single curvature bending the moment gradient factor for doubly symmetric members appears to give acceptable predictions of the critical load.

Keywords: Steel beam, Monosymmetric, Lateral-torsional buckling, Moment gradient factor

INTRODUCTION

The world is facing a climate emergency and this calls for innovative approaches as well as concerted effort in mitigating the increase in atmospheric carbon. This can be done through adoption of practices that foster reduction in anthropogenic carbon emissions or in carbon footprint in relation to construction related activities. For design engineers one way to reduce carbon footprint is to apply the practice of reuse in lieu of recycling. Recycling involves the use of energy and unless this is sourced from renewable energy it too potentially carries a significant carbon footprint. The reuse of structural members therefore provides a viable alternative for cyclic use of materials where the potentially more environmentally harmful options of disposal and introduction of new materials, or that of recycling using non-renewable energy sources are avoided. This is consistent with the options illustrated in Halliwell (2024) and discussed in part by Hayes (2024) showing the hierarchy of net zero design that can be adopted by design engineers. 'Net zero design' is a design approach that aims to mitigate increase in atmospheric carbon emissions by reducing the carbon footprint of structures based on proactive design decisions. This concept is illustrated in Figure 1 with the 'Build less' approach being achieved through consideration of repurposing, refurbishment and reuse of structural members.

The focus of this study is the reuse option. It has been shown by Mudenda and Zingoni (2022) that monosymmetric beams of the configuration shown in Figure 2 have the potential to be used for strengthening existing doubly symmetric I-shaped sections that need to have their flexural capacity or stiffness increased for reuse purposes. These sections exhibit some peculiarities including having a coincident shear centre and centroid at a given upstand height as well as a monosymmetry constant of zero at another upstand height. These are geometrical properties typically associated with doubly symmetric sections. These monosymmetric sections, for the simply supported case, also have a range over which they show increase in critical elastic moment. The point of peak value is observed to be closely related to the upstand height at which the shear centre and centroid are coincident. Beyond this point the critical moments starts to decrease with increase in upstand height. The shear centre movement also follows a peculiar path in comparison to an I-shaped monosymmetric section. This is discussed later.



Build nothing	Challenge the brief
Build less	Repurpose, Refurbish, Reuse
Build clever	Structural Configuration
Build efficiently	High utilisation/Carbon efficient materials
Minimise waste	Improve construction practices

Figure 1 Hierarchy of net zero design

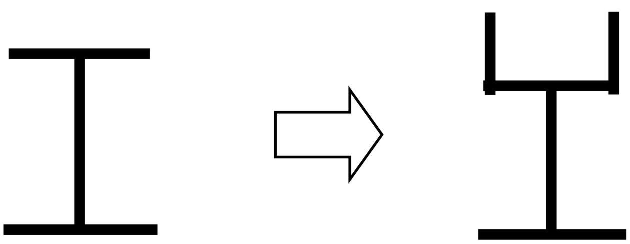


Figure 2 Doubly symmetric I-beam stiffened with flange upstands

Figure 2 shows how a doubly symmetric I-shaped steel beam is converted into a monosymmetric section by the introduction of flange upstand stiffeners. Conversion into this monosymmetric section has the potential to enhance flexural strength and stiffness of the section as discussed. This is desirable if the structure is repurposed in a manner that it needs to carry greater loads or if the member is to be used in a different structure where greater flexural strength or stiffness is needed. This can be adopted in lieu of replacing with a new I-shaped member of higher strength. Reuse in the same structure is particularly desirable as it obviates the need to incur costs of demounting the existing member, disposing of it and potentially replacing it with a new member that also may need costly scaffolding, manpower or machinery to mount, in addition to the carbon footprint of its production.

The use of such a monosymmetric section beam as the one considered may result in part or all of the member being subjected to a moment gradient that result in double curvature bending. For this case it is unclear whether the current guidance on the determination of the critical buckling load presented in design aids is valid. The study aims to verify such guidance provided in the South African Steel Construction Handbook (SASCH). This study is restricted to elastic critical buckling behaviour. In this preliminary study selected discrete members are investigated with a general approach to be explored in future studies on the basis of the observed outcomes.

Background

The use of monosymmetric sections for steel beams that do not have restraint to the compression flange is not so prevalent due, in part, to the complexity associated with the lateral-torsional buckling (LTB) behaviour of these sections. Lateral-torsional buckling is a stability failure that affects beams bending about their major axis and not having restraint to the compression flange. It is associated with a lateral movement of the compression flange in a direction perpendicular to the plane of loading and accompanied with twist. Early researchers such as Anderson and Trahair (1972) highlighted the '*Wagner effect*' which arises from the shear centre and geometric centroid not being at the same location for monosymmetric beams. This effect is encapsulated in the monosymmetry constant (β_x), a geometrical property associated with monosymmetric sections. A lot of the research on monosymmetric sections has focused on I-shaped sections, tee sections and compound sections made from I-sections with a welded channel cap. In order to find the strength of the beams, in most cases the critical elastic moment must first be determined. Codes of practice typically use the critical elastic moment equation for a uniform moment loading case and then allow for other loading or bending moment profiles by making use of a moment gradient factor. The uniform moment case is typically the only case that gives a closed form mathematical solution when the boundary conditions are pin type with warping of the cross-section permitted, hence its use. The moment gradient factor proves to be a convenient way of determining the critical load for those cases that are different to the uniform moment case with simple supports. However, the determination of moment gradient factors has largely been based on doubly symmetric sections making the direct extension to monosymmetric beams questionable and needing verification. Although, the South African national standard for design of hot-rolled sections (SANS 10162-1) does not provide any guidance on design of monosymmetric sections, the Southern African Steel Construction Handbook (SASCH) provides an equation for the elastic critical buckling moment of a monosymmetric section and implies that the same moment gradient factor as that for doubly symmetric beams can be used. The equation is shown in equation (1). A simplified equation for obtaining the monosymmetry constant has been developed by Kitipornchai and Trahair (1980). The actual equation and the simplified version for the monosymmetry constant (β_x) are shown in equations (2) and (3) respectively.

$$M_{cr,0} = \frac{\pi^2 EI_y\beta_x}{2L^2}\left[1 \pm \sqrt{1 + \frac{4}{\beta_x^2}\left(\frac{GJL^2}{\pi^2 EI_y} + \frac{C_w}{I_y}\right)}\right] \quad (1)$$

$$\beta_x = \frac{1}{I_x}\int_A y(y^2 + x^2)dA - 2y_0 \quad (2)$$

$$\beta_x = 0{,}9\, d'\left(\frac{2I_{yc}}{I_y} - 1\right)\left[1 - \left(\frac{I_y}{I_x}\right)^2\right] \quad (3)$$

In these equations d' is the distance between centres of area of the two flanges, I_{yc} being the second moment of area of the compression flange about its own strong axis [about y-axis in Figure 3 (c)], x and y being plate coordinates, and y_0 the distance between shear centre and centroid. It is worth noting that when equation (3) was presented, the authors stated that it has a validity range which depends on the second moment of area ratio and this particular version was derived on the basis of an I-shaped section. The monosymmetry constant for sections considered in the current study was obtained based on equation (2) so that validity is not violated. An approach presented by Hsu et al. (2012) was adopted and used to apply the integration.

Literature study

The most prevalent early form of the moment modification factor approach is reported to be that presented by Salvadori (1955). A simple modification by who to the equation for a uniform moment case ($M_{cr,0}$) to account for other load cases was proposed as shown in equation (4). The form of the moment gradient factor also known as the Equivalent Uniform Moment Factor (EUMF) was presented as in equation (5) with M_1 and M_2 being the moment values at the ends of the unbraced length under consideration. The end moment M_1 is taken as the one with the smaller value.

$$M_{cr} = C_b M_{cr,0} \quad (4)$$

$$C_b = 1.75 + 1.05\left({M_1}/{M_2}\right) + 0.3\left({M_1}/{M_2}\right)^2 \leq 2.3 \quad (5)$$

It was indicated by Suryoatmono (2002) that this equation appears in the 1986 edition of the American Institute of Steel Construction (AISC) code. Two versions of the EUMF have been presented by Ziemian (2010) with one of them being similar to equation (5). It is clarified here that the moment ratio (M_1/M_2), denoted by 'κ' in equation (6) is positive for double curvature and negative for single curvature. It was stated that these equations are applicable to linearly varying moments between brace points. This limitation is significant as in practice there are many cases where the bending moment will not vary linearly along the unbraced length.

$$C_b = 1.75 + 1.05\kappa + 0.3\kappa^2 \leq 2.56 \quad (6)$$

$$C_b = [0.6 - 0.4\kappa]^{-1} \leq 2.5 \quad (7)$$

It has been reported by Suryoatmono (2002) and Helwig et al. (1997) that Kirby and Nethercot (1979) presented an alternative equation for the EUMF which is applicable to both linear and nonlinear moment gradient diagrams between brace points. This equation is based on determining the maximum moment M_{max} in the unbraced span as well as moment values at quarter points (*A*, *B* and *C*) along the beam. This equation can be found in the 1999 versions of the AISC code in a slightly modified form as shown in equation (8).

$$C_b = \frac{12.5\, M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \quad (8)$$

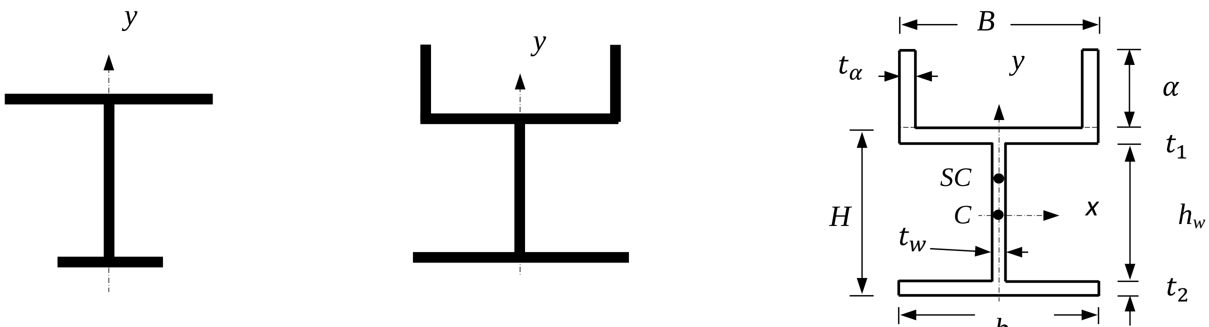
Many studies have been conducted to improve the accuracy of the EUMF equation, in particular to consider load height effects (when load is not placed at shear centre) as well as end conditions. So called 'quarter points' equations such as Equation (8) have been studied by Wong and Driver (2010) who proposed an improved version of this type of formula. The EUMF formula presented in the South African steel design code as well as the handbook is based on equation (6) and given as ω_2 but with the limiting value given as 2.5 instead of the 2.56 used here. It, therefore, appears that the South African design guides have not moved on from an equation meant for linearly varying bending moments to the more general 'quarter points' approach. The current study only focuses on the equations provided in the South African design guide documents.

MATERIALS AND METHODS

In order to determine the moment gradient factor, the critical elastic buckling moment for the uniform moment case was first obtained ($M_{cr,0}$). The bending moment was then altered to a case that differs from the uniform moment case and the critical moment determined, ($M_{cr,m}$). For example, a uniformly decreasing moment with maximum at one end and zero at the other end for the same length of the member. The ratio of the two moment values then gives the moment gradient factor as given in equation (9), with ω_2 being the moment gradient factor.

$$M_{cr,m} = \omega_2\, M_{cr,0} \quad (9)$$

Two sections were considered, one being an I-shaped monosymmetric section and the other being the stiffened beam which is the subject of the study. The I-section beam is studied to provide a comparison particularly because this member has been the subject of previous studies. The two different types of monosymmetric sections are shown in Figure 3 together with the geometric parameters used. For the Type 1 section, the upstand parameters in Figure 3 (c) are set to zero. The critical elastic buckling moment was determined using equation (1) with the moment gradient factor given by equation (6) based on the code of practice (SANS 10162-1) section for design of doubly symmetric beams. The analytical results were compared to finite element analysis (details of this is missing) results. This comparison was done to verify whether using the moment gradient factor for doubly symmetric sections for monosymmetric section behaviour predictions is a valid approach. The results are presented in tables and analysed to determine whether the use of the code equation gives safe or unsafe results for the section cases considered.



(a) Type 1 - MS (b) Type 2 – MS (c) Geometric parameters

Figure 3. Monosymmetric sections considered (a) and (b), and their geometric parameters (c)

Sections considered

The sections considered are shown in Table 1. Associated geometric properties based on Figure 3 (c) are given. An attempt was made to ensure that the members are compact so that they can attain the plastic moment, M_p. The initial slenderness parameter is given by:

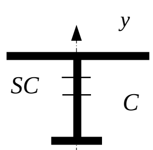
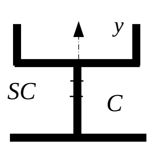
$$\bar{\lambda}_{LTi} = \sqrt{M_p / M_{cr}} \quad (10)$$

Initial slenderness parameters for the beam when subjected to the uniform moment case are selected such that the beam is slender i.e. the slenderness parameter is greater than unity (the point at which the plastic moment would match the elastic moment). In the slender beams the critical elastic moment is the critical value and therefore the beams are always likely to suffer from a global lateral-torsional buckling failure with no local buckling provided the plate elements are not slender. The degree of monosymmetry, ρ, is also considered for the selected members. This is given by equation (11). Member lengths vary as $\bar{\lambda}_{LTi}$ varies.

$$\rho = \frac{I_{yt}}{I_{yt}+I_{yb}} \quad (11)$$

with I_{yt} the second moment of area of the top flange and I_{yb} the second moment of area of the bottom flange about the y-axis of the section. The axis is shown in Figure 3 (c) or Table 1 figures.

Table 1. Members considered for study.

Member	Description								
Type - 1 	Beam:	B / b	H	t_1	t_2	t_w			ρ
	T1-1	150 / 75	400	10	8	5			0.91
	T1-2	150 / 132	400	10	8	7			0.65
	T1-3	100 / 75	250	8	5	4.5			0.79
Type - 2 	Beam:	B / b	H	t_1	t_2	t_w	t_α	α	ρ
	T2-1	55/55	100	5.7	5.7	4.1	5.7	10	0.65
	T2-2	55/55	100	5.7	5.7	4.1	5.7	50	0.84
	T2-3	146/146	251	8.6	8.6	6	8.6	75	0.79

*All dimensions in (mm)

Finite element analysis model

The software Abaqus (which version) was used for the finite element analysis to determine linear elastic critical buckling loads. The finite element model was calibrated against benchmark problems whose closed form analytical solution was available based on uniform moment loading. The boundary conditions were applied for the simply supported case with warping of the flanges allowed to occur freely. Rotation of the section was prevented at the end supports. These boundary conditions are shown in Figure 4 with U_i being the restrained displacement component. The longitudinal restraint U_z is applied at one end only (non-roller support end). A mesh convergence study was conducted to determine the optimum mesh size. An element size of 10 mm was found to give sufficiently accurate results and was adopted for the study.

RESULTS

Results for the two beam types for different beam slenderness and degree of monosymmetry values are presented in Tables 2 to 4. A typical buckled configuration of a monosymmetric beam is shown in Figure 5. The degree of monosymmetry was high for T1 and intermediate for T2 in the initial iteration given in Table 2. These were then interchanged for the study results presented in Table 3. The degree of monosymmetry for T2 could not be increased too much as this could result in slender stiffeners. The results show that the moment gradient factor from the design guides produces acceptable estimates for the single curvature bending case. However, once the member is in double curvature the accuracy of the moment gradient factor start to alter. The change is, in some of the observed cases, unconservative making this particularly undesirable.

Table 2. Results for Beams T1-1 and T2-1 for three moment gradient cases (FEM vs applied SANS 10162 code equation)

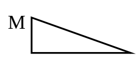
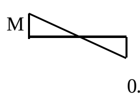
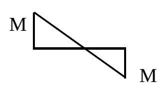
				FEM				Code		
Sketch of BMD		ρ		$\bar{\lambda}_{LT} = 1.22$	$\bar{\lambda}_{LT} = 1.61$	$\bar{\lambda}_{LT} = 2.00$		$\bar{\lambda}_{LT} = 1.22$	$\bar{\lambda}_{LT} = 1.61$	$\bar{\lambda}_{LT} = 2.00$
	T1-1	0.91	M_{cr} (kNm)	228.3	133.6	87.3		230.8	132.5	85.9
			ω_2	1.75	1.80	1.81		1.75		
	T2-1	0.65	M_{cr} (kNm)	17.8	10.1	6.5		18.2	10.5	6.8
			ω_2	1.82	1.80	1.80		1.75		
	T1-1	0.91	M_{cr} (kNm)	178.7	123.2	90.6		310	177.9	115.4
			ω_2	1.37	1.66	1.88		2.35		
	T2-1	0.65	M_{cr} (kNm)	24.4	13.7	8.7		24.4	14.1	9.2
			ω_2	2.49	2.45	2.42		2.35		
	T1-1	0.91	M_{cr} (kNm)	74.8	52.5	40.3		329.8	189.3	122.8
			ω_2	0.60	0.70	0.80		2.50		
	T2-1	0.65	M_{cr} (kNm)	22.9	13.6	8.9		26.0	15.0	9.8
			ω_2	2.34	2.43	2.47		2.50		

Table 3. Results for Beams T1-2 and T2-2 for two moment gradient cases (FEM vs applied SANS 10162 code equation)

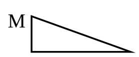
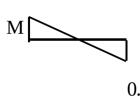
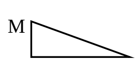
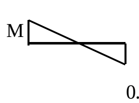
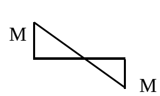
				FEM				Code		
Sketch of BMD		ρ		$\bar{\lambda}_{LT} = 1.22$	$\bar{\lambda}_{LT} = 1.61$	$\bar{\lambda}_{LT} = 2.00$		$\bar{\lambda}_{LT} = 1.22$	$\bar{\lambda}_{LT} = 1.61$	$\bar{\lambda}_{LT} = 2.00$
	T1-2	0.65	M_{cr} (kNm)	319.5	185.4	120.2		312.4	179.4	116.2
			ω_2	1.80	1.83	1.83		1.75		
	T2-2	0.84	M_{cr} (kNm)	32.3	18.0	11.4		31.2	17.8	11.6
			ω_2	1.78	1.80	1.78		1.75		
	T1-2	0.65	M_{cr} (kNm)	403.5	246.7	162.9		419.5	240.9	156.0
			ω_2	2.28	2.43	2.48		2.35		
	T2-2	0.84	M_{cr} (kNm)	42.3	24.1	15.1		41.8	24.0	15.5
			ω_2	2.35	2.41	2.36		2.35		

Table 4. Results for Beams T1-3 and T2-3 for three moment gradient cases (FEM vs applied SANS 10162 code equation)

				FEM				Code		
Sketch of BMD		ρ		$\bar{\lambda}_{LT} = 1.22$	$\bar{\lambda}_{LT} = 1.61$	$\bar{\lambda}_{LT} = 2.00$		$\bar{\lambda}_{LT} = 1.22$	$\bar{\lambda}_{LT} = 1.61$	$\bar{\lambda}_{LT} = 2.00$
	T1-3	0.79	M_{cr} (kNm)	83.4	48.0	31.1		86.8	47.1	30.4
			ω_2	1.80	1.83	1.83		1.75		
	T2-3	0.79	M_{cr} (kNm)	250.9	144.9	92.7		240.8	138.3	89.6
			ω_2	1.80	1.82	1.82		1.75		
	T1-3	0.79	M_{cr} (kNm)	98.4	60.7	40.7		116.6	63.2	40.9
			ω_2	2.13	2.31	2.39		2.35		
	T2-3	0.79	M_{cr} (kNm)	320.6	193.9	126.0		323.4	185.6	120.3
			ω_2	2.30	2.44	2.48		2.35		
	T1-3	0.79	M_{cr} (kNm)	55.4	36.2	25.9		124.0	67.3	43.5
			ω_2	1.20	1.38	1.52		2.50		
	T2-3	0.79	M_{cr} (kNm)	237.3	154.1	107.5		344.0	197.5	128.0
			ω_2	1.70	1.94	2.11		2.50		

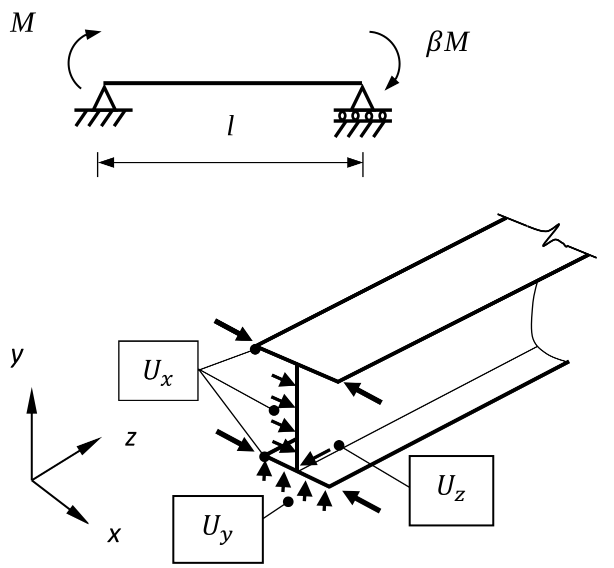


Figure 4. Boundary conditions in the finite element model

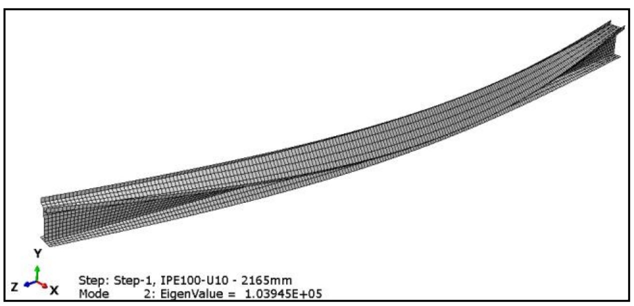


Figure 5. Buckling mode for a monosymmetric beam (Beam Type – 2)

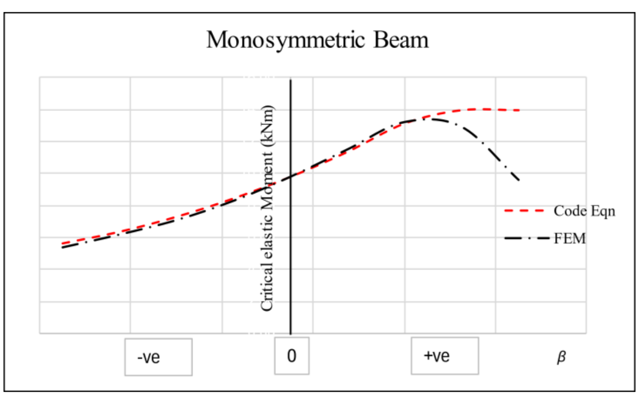


Figure 6. Critical elastic moment trend for code equation and finite element method (FEM) output

In the undesirable cases, the moment gradient factor from design guide equations produces critical moments that are greater than those predicted by the finite element models. Also, the moment gradient factor appears to change as the length of the member changes. This makes it particularly challenging to develop a single moment gradient factor. This is a challenge that has been noted by Knobloch et al. (2020) who are involved in developing the next generation of the Eurocodes. It is noted that the Type 2 section has a lower sensitivity than the Type 1 section for the cases where the loss in accuracy of the moment gradient factor was observed. However, it is seen that for results in Tables 2 and 4, in general once the member is in double curvature there is evident loss of accuracy in moment gradient factor predicted by the equation that is provided in the code SANS 10162-1 for doubly symmetric sections. This is illustrated in Figure 6.

These results indicate that the implied guidance in the Steel design handbook can produce unconservative results for some monosymmetric sections subjected to linear moment gradients between points of restraint that cause double curvature bending. It is important that when monosymmetric sections are used in cases involving double curvature bending between points of restraints, the critical elastic moment and subsequently member strength are determined for each particular case. A generalised moment gradient factor based on doubly symmetric sections should not be used. The 'Wagner effect' associated with monosymmetric sections can have an effect on monosymmetric sections when they are subjected to double curvature bending. This is due to the change in transverse torques arising from longitudinal stresses when the compression and tension flanges swap along the unrestrained length.

The difference in behaviour between the Type 2 and Type 1 sections in some of the observed cases might be due to the relative position of the shear centre in relation to the centroid. For the Type 1 section the shear centre position will always be above the centroid for the degree of monosymmetry above 0.5. For the Type 2 section it has been shown by Mudenda and Zingoni (2018) that the shear centre initially moves above the centroid but at a certain upstand height it reaches a peak height and then starts to move back down even meeting the centroid again. This gives a monosymmetric section that has a coincident shear centre and centroid at a given upstand height. The shear centre then moves below the centroid as the upstand height increases. This shear centre movement can counteract the adverse effects of the negative bending part of the double curvature and hence may reduce the sensitivity of this particular cross section. A more extensive parametric study still needs to be carried out to confirm these initial observations.

CONCLUSION

The current study has shown that using the moment gradient factor developed for doubly symmetric sections on monosymmetric sections subject to lateral-torsional buckling, and subjected to double curvature from linear moment gradient between points of restraint can produce unconservative predictions. For both beam types considered the design code equation for moment gradient factor produced similar outcomes as those from finite element analysis models for the single curvature case. For the case of double curvature there was generally a noticeable deviation from code equation predictions for some cases. It appears that in certain cases such as results from Table 4, beam type 1 showed more sensitivity in deviating from code predictions than the type 2 beam. Factors leading to this behaviour still need to be investigated further. It is, however, clear that there are some cases for which the code equation would produce unconservative predictions. It is recommended that for monosymmetric sections experiencing double curvature due to linear moment gradients between points of restraint the critical buckling loads must be determined for each particular case. The use of moment gradient factors that were developed for doubly symmetric beams must be avoided for these cases.

Selected symbols

G - Shear modulus
M_{cr} *– Critical elastic buckling moment*
M_p *– Plastic moment*
β *– End Moment ratio*
C_b, ω_2 *– Equivalent uniform moment factor/ Moment gradient factor*
I_x, I_y *– Major and minor axis second moment of area*
E - Elastic modulus

DECLARATIONS

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Data availability

The datasets used and/or analysed during the current study available from the corresponding author on reasonable request.

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Competing interests

The author declares no competing interests in this research and publication.

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